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HW Five, MTH 418, Spring 2016

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QUESTION 1. (i) Let H be a connected graph of order ≥ 4. Suppose that H has exactly three vertices that form the cycle C_3 but H has no other cycles. Convince me that $\chi(H) = 3$. [Hint: Consider the graph $D = H - C_3$.]. Is H a critical graph? explain.

(ii) Let H be a planar connected graph of order $n \geq 3$. Convince me that the number of all faces of H (bounded + unbounded) is $\leq 2n - 4$.

(iii) Let H be a planar connected graph of order 5 such that number of all faces is 6. Construct H. Is H a maximal planar?

(iv) Convince me that Q_4 is not a planar (Hint: show that Q_4 has a subgraph that is a subdivision of $K_{3,3}$). Now convince me that Q_n is not a planar for $n \geq 4$. [Hint: Only stare at the subgraph that you constructed for Q_4 and ... one more statement and you are done!]. Hence we conclude from this question that Q_n is planar only when n = 1, 2, 3.

(v) Let H be a planar connected graph with no triangles (i.e, no C_3 as a subgraph) and of order $n \geq 3$. Convince me that the number of all faces of H (bounded + unbounded) is $\leq n - 2$.

(vi) Let H be a connected maximal planar graph of order $n \geq 3$. Let D be a subdivision of H. Then it is clear that D is planar. Assume that every bounded face of D is C_4. How many vertices does D have? How many edges does D have? (give your answer in terms of n).

(vii) Let H be a connected 3-regular graph of order 10 (such graph is called Petersen Graph). Find $\chi(H)$, $\overline{\chi}(H)$. Find $\kappa(H)$. Is H a planar? Is H critical?

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